Assessing Household Welfare in Food Consumption: An Application of the Vartia Algorithm

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Abstract

This paper focuses on welfare assessment applied to the food sector, which is a relevant factor for Bolivian households. In 2022, Bolivia experienced stable inflation despite a generalized acceleration of prices worldwide, affecting real wages, purchasing power, and households' consumption baskets. The study quantified changes in economic welfare based on household food expenditures, taking price changes into account. We applied the Vartia algorithm to compute the Equivalent Variation (EV) and the Compensating Variation (CV). The EV and CV results were 1.19% and 1.64%, respectively. We also report estimates by income decile, department, and economic activity. This study informs economic and social policymaking—particularly minimum wage setting—by providing evidence on the increases required in response to rising food prices.

Keywords: *Microeconometrics, welfare economics, Vartia algorithm.*
JEL Codes: *C81, D6, C02.*

1 Introduction

The concept of welfare broadly refers to the resources and opportunities that people need to lead satisfying and productive lives [1]. Government actions can directly influence the population's welfare through social policies, investments in sectors such as health, education, housing, and the provision of basic services, as well as through wage increases, employment policies, and transfers to households, whether in cash or in kind.

In economic theory, two strands have been established for measuring overall or individual welfare. The first, initiated by Edgeworth [2], held that the measurement of utility was possible—akin to a modern-day hedonimeter capable of capturing an individual's mood.

By contrast, the second strand, proposed by Fisher [3], argued that welfare, being an unobservable variable, cannot be measured directly and must necessarily be derived from utility, indirectly inferred from observed behavioral choices.

Beyond measurement, there are factors that put a society's welfare at risk, the most relevant of which is the change in the price level, particularly food prices. This has a differentiated impact across socioeconomic strata—less pronounced among high-income individuals and more significant for the middle class and low-income groups.

At present there are techniques to measure the price effect within a basket of goods. In the microeconomic literature, five ways to measure welfare can be identified, developed by Hicks [4, 5]: change in consumer surplus, compensating variation, equivalent variation, Laspeyres variation, and Paasche variation.

In applied welfare analysis, it is common to employ metrics such as equivalent and compensating variation to evaluate and compare scenarios. These measures play a dual role: on the one hand, they gauge the income changes required to compensate for price variations; on the other, they classify budget changes in a manner consistent with consumer preferences.

This study uses the aforementioned measures in the analysis of economic welfare and public policy, with the aim of contributing to the current state of knowledge in this area. Section 2 examines the stylized facts related to inflation, food inflation, household consumption by dimension and the relevance of food within the consumption basket, and wage policy, which

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governments apply to preserve the purchasing power of wages.

Section 3 reviews the welfare literature, theorizing the measurement and description of equivalent and compensating variation in income, thereby establishing a solid theoretical framework for the empirical study.

Section 4 provides a brief description of the process of information gathering and processing used in the empirical framework and in the application of the Vartia algorithm. Section 5 presents the results obtained from applying the Vartia algorithm to the case of Bolivia. It also reports equivalent and compensating variation by department, income decile, and economic activity.

Finally, the conclusions derived from the study are presented. The main findings are summarized, and possible avenues for further research in the field of measuring economic welfare in Bolivia—an area little explored by researchers at the microeconomic level—are suggested.

2 Stylized Facts

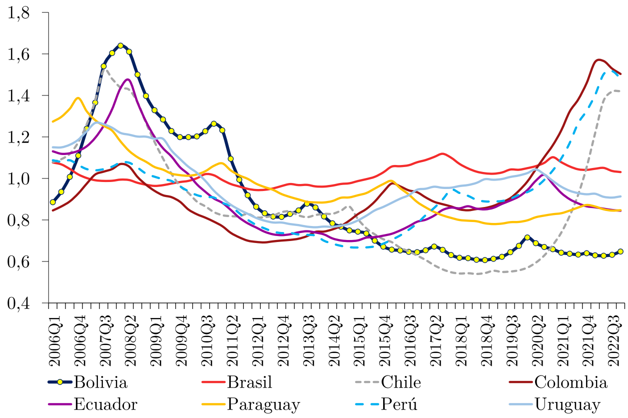
In 2020, the COVID-19 pandemic had a negative impact on the world economy, manifesting itself in economic and social setbacks, with declines in output and productivity, rising unemployment, and increases in the price level.

The emergence of new variants further heightened global uncertainty, leading to stricter health restrictions to contain the spread of the virus. This resulted in a contraction of the supply of goods. Subsequently, the geopolitical tensions of 2022 in Europe escalated into an armed conflict between Russia and Ukraine, disrupting the supply of agricultural products, fuels, minerals, and fertilizers. Transportation costs also rose, developments that led to an acceleration of inflation worldwide.

The inflationary process caused by these events affected South America, especially in 2022, increasing price volatility in food items, which in turn resulted in a decline in real income and a higher cost of living, with adverse effects on welfare overall.

Volatility is a crucial factor that influences price stability and households' ability to access a consumption basket. Notably, countries with lower price volatility enjoy greater stability, which strengthens households' confidence in their ability to acquire goods. In contrast, countries with higher volatility face greater challenges in maintaining price stability, making it more difficult for households to access a basket of goods. Figure (1) shows the volatility[1] of food prices in countries of the region during the period 2006–2022. The measure of food price volatility provides information on price stability and its impact on households' economic welfare. An analysis of average estimated volatility shows that Bolivia, Chile, and Ecuador have low levels of volatility in food and beverage prices, whereas Brazil, Uruguay, and Paraguay exhibit high instability in prices for this category.

Figure 1: Volatility of food inflation in South America, 2006q1–2022q4



Prepared by the authors.
Source: Economic Commission for Latin America and the Caribbean (ECLAC).

In Bolivia, headline inflation rose from 0.9% in 2021 to 3.1% in 2022, while food inflation increased from 0.5% to 5% over the same period; see Figure (2). However, this inflationary process does not compare to that of other countries in the region, where price level increases were more pronounced.

For the food and non-alcoholic beverages division, the South American region experienced higher inflation in Argentina (4.7%) and Colombia (2.7%), as a result of the international context, with a more pronounced upward trend visible from the last quarter of 2021 in Figure (3). At the opposite end, Bolivia and Paraguay recorded deflation of 0.2% and 0.9%, respectively.

Inflation has a negative impact on individuals' welfare, especially among low-income or poor populations, as it shapes their consumption of goods and services, with the food dimension carrying a higher weight in household budgets.

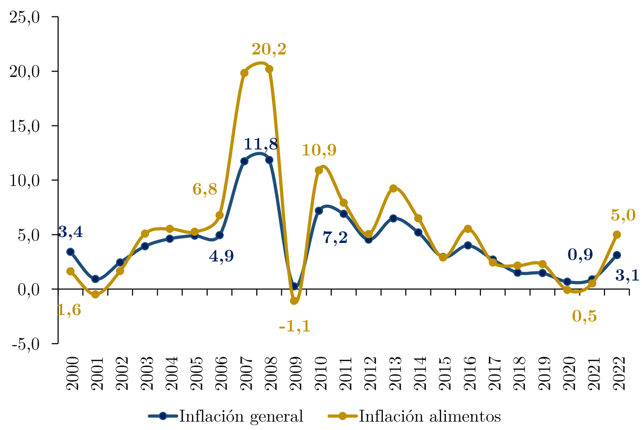
[1] A standard stochastic volatility (SV) model was used, described below:

$$y_t = \mu + \varepsilon_t^y \qquad \varepsilon_t^y \sim N(0, e^{h_t}) \tag{1}$$

$$h_t = \mu_h + \psi(h_{t-1} - \mu_h) + \varepsilon_t^h \qquad \varepsilon_t^h \sim N(0, \omega_h^2) \tag{2}$$

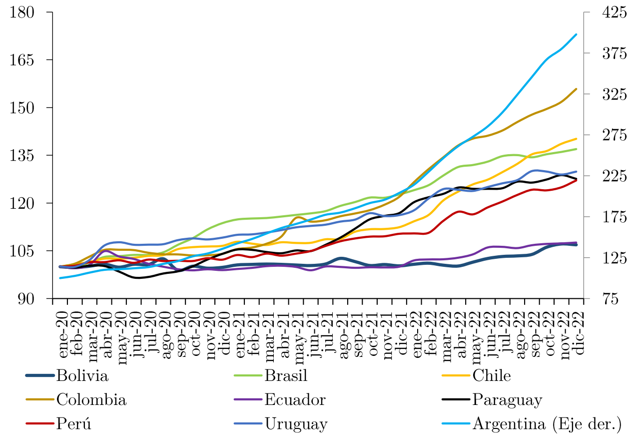
Where log-volatility, represented by h_t, follows a stationary $AR(1)$ process with $|\psi_h| < 1$ and unconditional mean μ_h. The shocks ε_t^y and ε_t^h are uncorrelated. For further details, see Chan and Grant [6], Yang and Hamori [7], and Fernández and Rodríguez [8].

Figure 2: Headline and food inflation in Bolivia, 2000–2021



Source: National Institute of Statistics (INE).

Figure 3: Price index for Food and Non-Alcoholic Beverages in South America (2020=100), 2020–2022



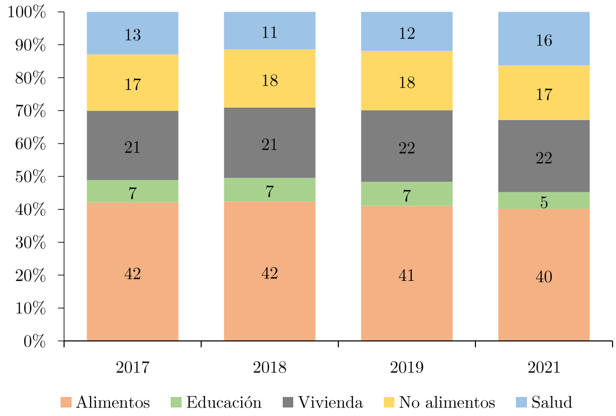
Source: National statistics institutes of each country.

Indeed, Bolivian households allocate a large share of their budgets to food; in 2021 it represented 40% of total expenditure, followed by housing (22%), non-food (17%), health (16%), and education (5%), as shown in Figure (4).

The importance of food is also reflected in the CPI basket: of the 397 products and services, 27% correspond to food and non-alcoholic beverages, and 14% to food and beverages consumed outside the home, for a total of 41%. An analysis of inflation dynamics and the contribution of its divisions shows that, of the annual 3.1% inflation rate, 2.2 percentage points (pp) correspond to food, equivalent to 68%.

However, one way to offset the effects of inflation on households is through wage policy. This measure is implemented by many countries in South America under different names. In Bolivia, Chile, Brazil, and Uruguay it is called the national minimum wage; in

Figure 4: Household expenditure by dimension, 2017–2021



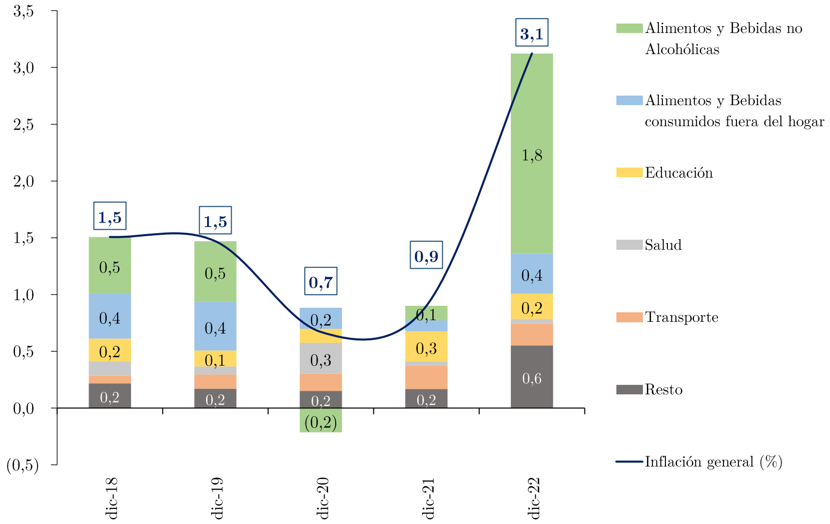
Source: National Institute of Statistics (INE) of Bolivia, based on Household Survey.

Argentina it is the vital minimum wage; in Colombia, the legal minimum wage; in Ecuador, the unified wage; and in Peru, the vital minimum remuneration.

In Bolivia, the national government increases the national minimum wage taking the previous year's inflation as a reference point. Since 2006, the country has applied this measure more consistently, as it is a way to restore the purchasing power of the national currency; in this sense, the national minimum wage rose from Bs440 in 2005 to Bs2,250 in 2022. Compared with inflation, the 2022 wage increase (4%) exceeded the previous year's inflation (0.9%), representing a 3.1 pp increase in the real minimum wage.

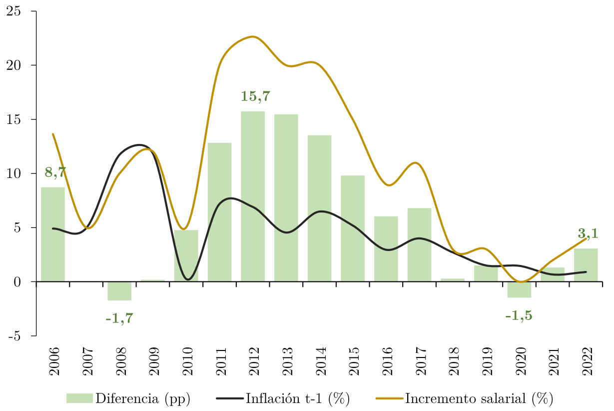
Wage policy plays a fundamental role in South America, although its trajectory has not been constant over time in many countries due to various internal and external factors. Bolivia, however, experienced a rapid increase in the national minimum wage starting in 2005, narrowing existing gaps in this indicator relative to other countries. In 2000, Bolivia had the lowest national minimum wage in the region at US$54, while Argentina had the highest at US$200. By 2022, the situation had reversed: Bolivia, with a minimum wage of US$323, moved up in the rankings and surpassed Argentina (US$312), partly due to the crisis affecting the Argentine economy, high inflation, and the continual depreciation of its currency. This reshaped the positions of minimum wages in the region, with Uruguay posting the highest minimum wage at US$471 and Brazil at the opposite end with US$235.

Figure 5: Contribution of divisions to Bolivia's annual inflation, 2018–2022



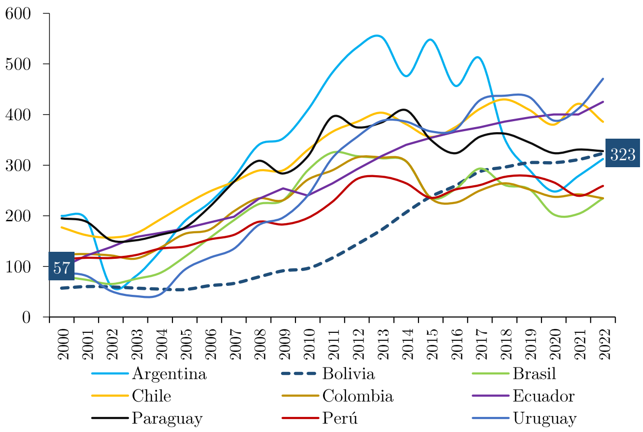
Source: National Institute of Statistics (INE) of Bolivia.

Figure 6: Increases in the national minimum wage and inflation in Bolivia, 2006–2022



Prepared by the authors.
Source: National Institute of Statistics (INE) of Bolivia, Social and Economic Policy Analysis Unit (UDAPE), and Supreme Decrees of each year.

Figure 7: National minimum wage in South America, 2000–2022 (US dollars)



Prepared by the authors.
Source: National Institute of Statistics (INE) of Bolivia, Social and Economic Policy Analysis Unit (UDAPE), and official institutes of each country.

3 Literature Review

Economics plays a key role in providing *policymakers* with accurate information on policy impacts. In this regard, changes in prices are highly relevant to household welfare. These changes manifest themselves in the consumer market (goods prices) or the government sector (taxes and subsidies), which can affect household income and its distribution. In addition, prices are affected in financial markets (interest rates) and in labor markets through wages.

These issues are fundamental to understanding and evaluating the impact of prices on welfare. That is, any change in the price of a good can have a significant effect on the quantity consumed, which in turn can modify the level of utility obtained from that good. The ideal measure for assessing the change in welfare would be one that reflects the variation in utility experienced as a result of a specific economic policy.

Consider two sets of prices and income, (p^0, m^0) and (p', m'), which represent different economic policy regimes. If (p^0, m^0) is the initial state and (p', m') is the change in the budget, the welfare variation would be the difference between indirect utilities, that is, $v(p', m') - v(p^0, m^0)$. If this utility difference is positive, a change in economic policy for the consumer would be justified; otherwise the change would not be advisable.

Although the utility approach is purely ordinal[2], a monetary measure of welfare is useful to policymakers when assessing the benefits and costs that would affect consumers. Therefore, the money-metric indirect utility function is an appropriate measure for evaluating the utility difference and the impact on welfare. [9]

In the analysis of money-metric indirect utility, two measures are used—equivalent variation and compensating variation[3]—to evaluate utility differences, as proposed by Hicks [4, 5]. Equivalent variation is based on current prices and seeks to determine what change in income at these prices would be equivalent to the proposed variation in terms of its impact on utility. Compensating variation, on the other hand, is based on the new prices and seeks to determine how much the consumer's income should change to compensate for the price change and keep utility constant.

3.1 Equivalent and Compensating Variation in Income

The objective of welfare economics is to evaluate the impact of economic policies and projects on people's well-being. To this end, standard measures such as equivalent and compensating variation in income are used to determine how changes affect welfare relative to the available alternatives.

As Antelo [11] notes, a change in welfare can occur when a consumer, faced with two consumption bundles x^* and $x^\blacklozenge$, where $x^* \succ x^\blacklozenge$, nevertheless chooses $x^\blacklozenge$ because x^* is unattainable. If a change in the value of some parameter allows the consumer to choose x^*, then welfare will increase. In other words, this change allows the consumer to attain a consumption bundle that was more desirable.

Consider two price vectors, p^0 and p^1, corresponding to an initial and final situation. We assume that income, m, is the same in both situations. To determine whether the consumer is better or worse off, we can compare the utilities obtained in each situation, $u(x^0(p^0, m))$ and $u(x^1(p^1, m))$. For this, we use Marshallian demands and compare the indirect utilities $v(p^0, m)$ and $v(p^1, m)$.

Although it is possible to compute utility differences using the indirect utility function, these differences are not directly observable because utility itself is not observable; in other words, the utility function is ordinal. [11]

To express utility differences in a quantitatively measurable magnitude (monetary terms), we use the expenditure function, defined as $e(p, u) = px^h(p, u)$, where $x^h(p, u)$ is the Hicksian demand system. Because this function is a dual representation of utility and its variations, we can use it to measure utility differences in monetary terms.

Measuring Welfare through Monetary Valuations. The measurement of consumer welfare in monetary terms has been widely studied in the literature, as discussed in Hicks [12], Chipman and Moore [13] and Willig [14].

It has been recognized in the literature that the direct utility function (DUF) cannot measure the cardinal[4] amount of the welfare change, since utility itself is unobservable. However, it is argued that the expenditure function (the inverse function of indirect utility), which is a monotonic increasing transformation of the DUF, can measure welfare changes in monetary terms. This allows us to assess the welfare change produced by variations in prices and consumer income.

Consider two periods, $t = [0, 1]$. We examine the variation in consumer utility in period 1 relative to period 0 when the price of a good changes. Let $u^R \equiv v(p^t, m)$ be a reference utility level and the expenditure functions $e(p^t, v(p^t, m)) \equiv e(p^t, u^R)$. These functions indicate the minimum income required to reach, at current-period prices t, p^t, the reference utility level u^R. The difference $e(p^1, u^R) - e(p^0, u^R)$ represents the welfare variation measured in monetary terms with respect to the utility level u^R when prices change from p^0 to p^1.

When measuring changes between two periods, it is important to define a reference level so that the measure is coherent. Two reference levels are used, resulting in two welfare measures: compensating variation in income if u^0 is used as the reference level and equivalent variation in income if u^1 is used.

Compensating Variation in Income In this case, the reference utility level is that of the initial period, u^0, which yields the following measurement of the welfare change.

- **Compensating Variation in Income (VCR)[5]:** This is the amount of money that must be given to or taken from the consumer to maintain the initial utility level (u^0) in the final situation after the increase or reduction in prices.

[2] This refers to a way of ranking or comparing consumer preferences in terms of their order of preference rather than assigning a specific numerical value.

[3] These are reasonable measures for assessing how changes in prices affect welfare. Both measure the differences between utilities generated by price changes, but their magnitudes may differ because they are based on different reference utility functions. Nonetheless, it is important to note that the sign of both measures is always the same. [10]

[4] A numerical measure used to quantify a consumer's satisfaction, allowing comparisons of the level of preference across different consumption options.

[5] Alternatively, VCR is defined as the amount of money that, when added to or subtracted from the consumer's income, keeps him or her at the initial utility, u^0, when prices change.

Using the expenditure function, if $e(p^1, u^0)$ is the minimum income, at prices p^1, to reach the utility level u^0, with $u^0 = v(p^0, m)$, while $e(p^0, u^0)$ is the minimum income, at prices p^0, to achieve the utility level u^0, then VCR is expressed as:

$$VCR(p^0, p^1, u^0) = e(p^1, u^0) - e(p^0, u^0) \qquad (3)$$

rewriting the previous equation:

$$e(p^0, u^0) = e(p^1, u^0) \pm VCR(p^0, p^1, u^0) \qquad (4)$$

We see that VCR is a negative amount if the consumer is worse off with the price change, which occurs when $p^1 \geq p^0$, in which case $v(p^1, m)$ is less than $v(p^0, m)$ and income must be given to keep the initial utility. Indeed, since $e(p, u)$ is non-decreasing in p, when prices increase in period 1, $e(p^1, u^0) - e(p^0, u^0) > 0$, so that in $t = 1$ it is necessary to spend more than in $t = 0$ to maintain utility u^0. This is why VCR is negative[6]. The opposite occurs when $p^1 \leq p^0$, in which case the consumer is better off with the price change and must spend less to maintain utility u^0, so VCR is positive given the inverse relationship between expenditure and utility.

Formally, we have $v(p^1, m - (-VCR)) = v(p^0, m)$ if $p^1 \geq p^0$ (price increase) and $v(p^1, m - VCR) = v(p^0, m)$ if $p^1 \leq p^0$ (price decrease). The content of this expression, in the case $p_1^1 > p_1^0$, is illustrated in Figure (8). We observe that when the price of good 1 increases, VCR is given by the amount of income BA, which is negative. This means that if the consumer is not provided with that additional amount of income, the price increase will result in a welfare loss of that amount.

Using the fundamental theorem of calculus together with Shephard's lemma, VCR is obtained by computing the integral of the Hicksian demand corresponding to utility level u^0 between the prices that changed between $t = 0$ and $t = 1$. That is, VCR is calculated as:

$$VCR(p^0, p^1, u^0) = e(p^1, u^0) - e(p^0, u^0)$$
$$\int_{p_1^0}^{p_1^1} \frac{\partial e(p, u^0)}{\partial p_1} dp_1 = \int_{p_1^0}^{p_1^1} x_1^h(p, u^0) dp_1 \qquad (5)$$

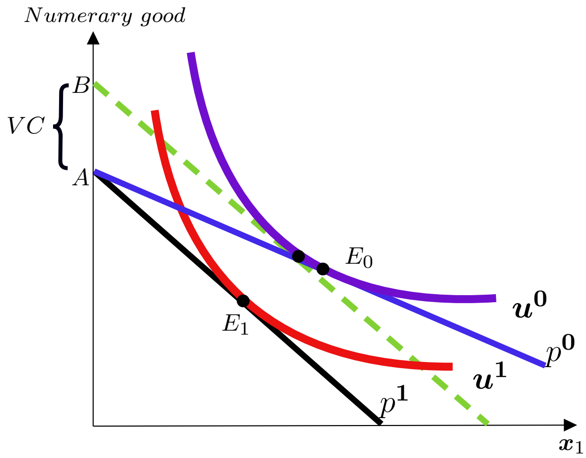
Equivalent Variation in Income Now consider the final period's utility level as the reference, u^1. This yields the following definition:

- **Equivalent Variation in Income (VER)[7]:** This is the amount of money that must be given to or taken from the consumer, at the initial situation's prices and income, so as to reach the same utility level as in the final situation, u^1.

[6] The expenditure function is the inverse of the indirect utility function, so if we must spend more to maintain the utility level, VCR will be negative, and vice versa.

[7] VER can also be defined as the amount of income that makes the price change from p^0 to p^1 have the same effect on the consumer's welfare as if income had changed from m to $m \pm VER$.

Figure 8: Compensating Variation in Income



Note: For an increase in the price of x_1
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Formally, we have:

$$VER(p^0, p^1, u^1) = e(p^1, u^1) - e(p^0, u^1), \qquad (6)$$

where $e(p^1, u^1)$ is the minimum income to reach utility level u^1 when prices are p^1 and $e(p^0, u^1)$. If we rewrite (6) as:

$$e(p^0, u^1) = e(p^1, u^1) \pm VER(p^0, p^1, u^1), \qquad (7)$$

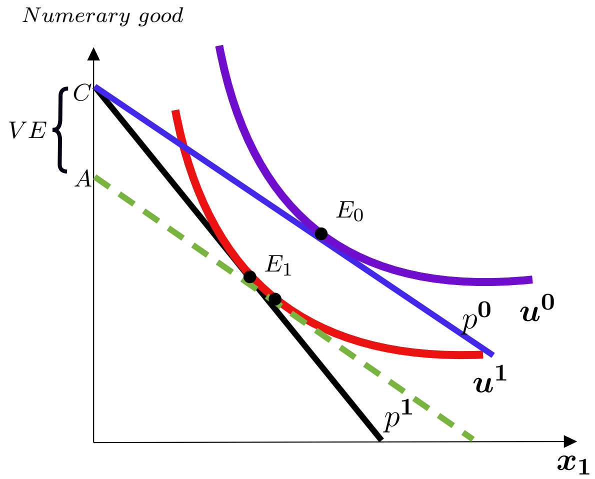
we verify that VER is positive when the price change is such that $p^1 \leq p^0$ and improves consumer welfare, whereas it is negative when $p^1 \geq p^0$ and welfare worsens.

That is, VER is the income that makes $v(p^0, m - (-VER)) = v(p^1, m)$ if $p^1 \geq p^0$ and $v(p^0, m - VER) = v(p^1 \leq p^0$. In Figure (9) we see that, in response to an increase in the price of good 1, $p_1^1 \geq p_1^0$, VER is given by the amount of money CA. Thus, VER translates into an integral over the Hicksian demand corresponding to utility level u^1.

$$\text{VER}(p^0, p^1, u^1) = e(p^1, u^1) - e(p^0, u^1)$$
$$= \int_{p_1^0}^{p_1^1} \frac{\partial e(p, u^1)}{\partial p_1} \, \mathrm{d}p_1 = \int_{p_1^0}^{p_1^1} x_1^h(p, u^1) \, \mathrm{d}p_1 \qquad (8)$$

When comparing two situations in terms of welfare and their relationship with price variation, we may observe an improvement (worsening) in the consumer's situation. Both VCR and VER have a positive (negative) sign, as appropriate. However, the magnitudes of these measures may differ because they consider different utility levels. With respect to consumer welfare, the following holds:

Figure 9: Equivalent Variation in Income



Note: Case of an increase in the price of x_1
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- *When $p_1^1 > p_1^0$, the reduction in welfare is such that, in absolute value, $VCR > VER$ if the good is normal and $VCR < VER$ if it is inferior.*
- *When $p_1^1 < p_1^0$, the increase in welfare is such that, in absolute terms, $VCR < VER$ if the good is normal, whereas $VCR > VER$ if it is inferior.*

The choice of the appropriate measure depends on the specific problem to be addressed. To establish compensation for new prices, compensating variation is appropriate. However, if the objective is a willingness-to-pay measure, equivalent variation is probably better; this measure evaluates the income variation at current prices, which facilitates the assessment of monetary value relative to hypothetical prices. In addition, when comparing policy change proposals, equivalent variation keeps prices fixed, whereas compensating variation uses different price levels.

Regarding the relationship between equivalent variation (VER) and compensating variation (VCR), specifically in the context of a single price change and a normal good, VER will be greater than VCR for a price decrease and VER<VCR for a price increase. This relationship changes when the good in question is inferior. If there are no income effects for a particular good, then VER and VCR will be the same for a given price change. [10]

When comparing two monetary measures of welfare, the question arises as to whether both yield similar results. However, these two measures do yield different results. Only in the case where the income effect in Slutsky's equation is zero will both measures coincide ($AB = AC$). This result is explained by the fact that, in the absence of an income effect, the indifference curves are parallel to each other, which implies that any change in income will not alter the quantity consumed of the good.

3.2 Partial Equilibrium

Within microeconomic theory, it is important to highlight the two welfare theorems in relation to a competitive general equilibrium. The first theorem states that when a competitive equilibrium is reached with a set of prices and an allocation, the latter is Pareto efficient. The second theorem states that if an allocation is efficient, then there exists a distribution of wealth among agents that is part of a competitive equilibrium.

However, in general equilibrium there are distortions such as market failures, asymmetric information, incomplete markets, externalities, among others. These factors prevent market clearing. Moreover, the task of addressing welfare is complex and even more difficult to quantify in empirical evidence. Therefore, in the present study partial equilibrium will be considered; this approach allows us to examine a particular market that is directly affected by the policy change, assuming that income effects are marginally small and that the prices of other goods are not affected by what happens in this market.

The partial-equilibrium approach offers several technical advantages by requiring less information and enabling simpler welfare evaluations. It also allows incorporating phenomena not considered in the assumptions, such as externalities and imperfect competition. In this paper, it is appropriate to adopt a partial-equilibrium approach, in line with Rubin de Celis and Espinoza [15].

3.3 Measuring Welfare

The measurement of welfare is a central topic in economic theory and has generated a wide variety of approaches to its evaluation. In particular, the equivalent and compensating variation approach comprises standard measures widely used to quantify changes in consumer welfare. These methods are fundamental to understanding and measuring the impact of prices on individuals' welfare.

Hicks [16] described five measures for quantifying welfare changes under price variations in the field of applied welfare economics, including the change in consumer surplus (CS)[8], equivalent variation in income (VER), compensating variation (VCR), Laspeyres variation (LV), and Paasche variation (PV). In this article,

[8] It has been criticized by some authors, who argue that (1) it is only valid under certain conditions, such as the invariance of the marginal utility of real income; (2) it does not take into account distributional changes that may result from price changes; (3) it focuses on partial equilibrium; (4) it may be unsuitable for large price changes; and (5) it does not consider the axiom of revealed preferences. [17]

we address the equivalent and compensating variation in income.

In general, to address the integrability problem (a system of partial differential equations in terms of the expenditure function) and the measurement of welfare changes (the difference between the expenditure function across two different periods), numerical approximation methods are used that consist of recovering compensated income (expenditure) from the ordinary demand function.

Breslaw and Smith [18] developed an efficient method to estimate an accurate measure of welfare in a system of n equations, along with the variance. The quadratically convergent algorithm is distinguished by its accuracy and ability to estimate both equivalent and compensating variation. Instead of applying numerical integration techniques directly, the authors chose to represent the expenditure function as a Taylor expansion, based on the approximation of McKenzie and Pearce [19]. This algorithm made an important contribution to the applied welfare literature because it provides an exact estimate of equivalent and compensating variation in a system of n equations, in addition to estimating the corresponding confidence intervals.

Because the true utility and demand functions are unknown, estimates of welfare changes are often based on approximations. For the purposes of this paper, we apply the numerical approximation proposed by Vartia [20]. The Vartia algorithm is a rigorous computational method for estimating equivalent and compensating variation in the demand for goods. Vartia's methodology has been used in multiple empirical studies aimed at evaluating consumer welfare and the effects of public policies.

The approach of Vartia [20] to measuring welfare changes differs from traditional approaches in economics, which start from an ex-ante utility function or derive welfare changes through a Taylor expansion of an unknown utility function. Instead, Vartia proposes a revealed-preference-based approach that works backward, starting from a known demand function and deriving the change in utility from this function. In his work, Vartia derives the conditions linking the demand function $h(.)$ to the indirect utility function $V(p, C)$ and uses these conditions to derive the first-order differential equation in the money-income (expenditure) function $C(.)$.

$$\frac{\partial C(t)}{\partial t} = \sum h_k(p(t), C(t)) \frac{\partial p_i(t)}{\partial t} \tag{9}$$

Where t refers to the iteration number. With prior knowledge of the demand function, it is possible to derive x iteratively and approximate ΔC. Therefore, VC can be expressed in terms of the sum of marginal changes over a known demand function.

$$VC = \frac{1}{2} \sum_t \sum_k \left(x_k(p^t, m^t) + x_k(p^{t-1}, m^{t-1})\right) \times \left(p_k^t - p_k^{t-1}\right) \tag{10}$$

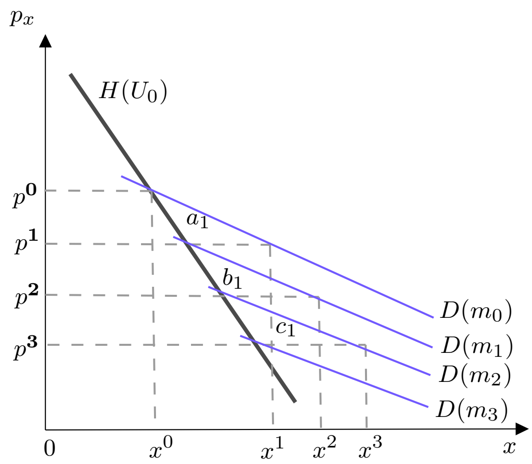
For the VC measure we have $p^{t=0} = p^a$ as the initial price and $p^{t=n} = p^b$ as the final price when the number of iterations is n, and $p^t - p^{t-1} = \frac{dp}{n}$. The Vartia algorithm is reversible and this allows estimating VE at price p_t as:

$$VE = \frac{1}{2} \sum_t \sum_k \Big(x_k(p^{n-t}, m^t) + x_k(p^{n-(t-1)}, m^{t-1})\Big) \cdot \Big(p_k^{n-t} - p_k^{n-(t-1)}\Big) \tag{11}$$

and $m^t = \hat{m}^{t-1}$. Where p_k^{n-t} for $t = 0$ is the price of good k in the final period.

The Vartia algorithm and its application to the estimation of compensated income m^c are illustrated in Figure (10), where the price decreases from p^0 to p^3.

Figure 10: Vartia Algorithm for Computing VC



Note: Case of a reduction in the price of x
Source: Extracted from Chen [21]

To compute compensated income at p^1, the initial income must be added to area a_0. However, this latter is simply approximated by $a_0 + a_1$ by using the Marshallian demand function as an intermediate step in the estimation. After this first iteration, we move to the second iteration to evaluate compensated income at p^2. This is based on the estimated m^1 plus the VC from p^1 to p^2, which is approximated by area $b_0 + b_1$. The same procedure is repeated to evaluate m^3 at price p^3. In the end, compensated income (m^3) at p^3 equals $(m + (a_0 + b_0 + c_0) + (a_1 + b_1 + c_1))$. Of course, the error term $(a_1 + b_1 + c_1)$ converges to zero when the number of iterations is large and the price change in each iteration becomes infinitesimal. In this way, it is possible to use the Vartia algorithm to determine the relationship between price changes and the variation in compensated income.

The Vartia and Breslaw–Smith (BS) algorithms are commonly used approaches to measure changes in economic welfare. According to Sun and Xie [22], who analyzed the accuracy of these methods, the Vartia algorithm and the BS algorithm are numerical methods for measuring compensated income (expenditure) and converge quadratically. The authors conduct different simulations and find that when the price change within each partition step is small, the error of the Vartia algorithm is approximately half that of the BS algorithm, and conclude that the Vartia algorithm is more accurate than BS.

To assess welfare measures and computational methods, Araar and Verme [17, 23] conducted a comparative study of the different welfare measures and concluded that, in general, the welfare measures converge to similar results for price changes below 10%, regardless of demand parameters. However, when price changes exceed the 10% threshold, the measures begin to diverge significantly. This finding has important implications for welfare evaluation under different price-change scenarios and highlights the need to carefully consider the effects of price changes in measuring economic welfare.

At the national level, a notable study applies Vartia's method to microeconomic data in the education sector. In particular, we take as a reference the work of Rubin de Celis and Espinoza [15], which analyzes welfare changes through equivalent variation applied to primary education in Bolivia using two methods: integrability and the Vartia algorithm, under a partial-equilibrium approach. The authors compute equivalent variation assuming a 1% decrease in the price of primary education and specifying a linear and a log-linear demand for each department in Bolivia.

3.4 Demand Function Specification

Following Border [24], we postulate a linear demand function which, by integrability, has its origin in a Cobb–Douglas utility function; this specification is the most widely used in documents related to welfare measurement. The integrability problem consists in recovering the consumer's utility function from the demand function; the implications of this are significant in terms of the well-behaved nature of a demand function, and the proof of integrability is described in Annex F.

It is important to keep in mind that if we consider a system of demand functions $x(p,m)$ that maximize the consumer's utility, we can summarize observable behavior as follows: the balanced budget condition holds, the Slutsky matrix associated with $x(p,m)$ is negative semidefinite, and $x(p,m)$ is symmetric. These conditions are fundamental to ensuring that the demand function is homogeneous of degree zero in prices and income. Thus, if a function that depends on prices and income satisfies the three conditions mentioned above, we can assert that the demand function is continuously differentiable, well-behaved, and exhausts the consumer's available income.

4 Data

Information from household consumption, demand, or welfare surveys is infrequently available in terms of both access and construction. Nevertheless, we deemed it appropriate to use Bolivia's 2021 Household Survey (EH) from the National Institute of Statistics (INE), since the data collection adheres to international statistical standards, yielding a nationally representative and reliable sample; therefore, the data quality supports the results reported in this study.

The 2021 Household Survey (EH) collects social, economic, and demographic characteristics at the national level. First, we considered the food module, which gathers household consumption information disaggregated into 75 products, including bread and cereals, fruits, sugar, meats and cold cuts, dairy products and eggs, vegetables and tubers, and oilseeds. This survey includes a sample of 963,525 observations from 12,847 families; however, not all households consume all 75 products. Consequently, we retain only households that consume at least one of these 75 products, resulting in a study sample of 251,780 observations. We also used the persons module to identify demographic, educational, and social characteristics, among others.

Given the characteristics of the Bolivian market—where units of measurement differ—we standardized all products to the unit "kilograms," using an average weight for unit items (e.g., bread) or applying density for liquids.

For prices, we mapped the 75 household consumption products to the Consumer Price Index (CPI) basket from INE, and then constructed a price index for each product. With this information, we additionally produced indices by city or conurbation, product, division, and for foods and non-foods. We also incorporated the CPI weights for each product.

5 Results

5.1 Food Demand Model

In this section, we estimate the quantity consumed by each household head across the 75 products in the "food and beverages" division. For product prices (p_i), we use the 2021 product-specific price index for each of the nine departments.

Disposable income is measured using household income ($yhog_i$), and we include control variables (x_i) to adjust the model for factors such as household size, age of the household head, schooling, among others. The model specification follows a linear demand, in the spirit of Soldán and Villarroel [25], Nogales [26], Rodrigues Torrez [27], and primarily Rubin de Celis and Espinoza [15]. The linear demand equation used to estimate the relationship between the quantity demanded of each product and its price and income is:

$$cant_i = \alpha + \beta * p_i + \gamma * yhog_i + \theta * x_i + \epsilon_i \quad (12)$$

Here, α is the constant, β_i is the demand slope for product i, γ is the income elasticity of demand, θ is a vector of coefficients on the control variables, and ϵ_i is the error term.

Econometric Estimation of Demand. Econometric analysis can face potential challenges—such as omitted variables and simultaneous causality—that generate correlation between the error and the regressors (e.g., $E(u|x) \neq 0$), rendering the OLS estimator inconsistent and biased. As a consequence, the marginal effect of an exogenous change in the jth regressor x_j cannot be interpreted, so instrumental variables are employed [28, 29].

If the instrument (Z_i) is relevant, then variation in the instrument is related to variation in X_i. If, in addition, the instrument is exogenous, then the portion of variation in X_i captured by the instrument is exogenous. Therefore, an instrument that is both relevant and exogenous can capture exogenous movements in X_i. This exogenous variation can then be used to estimate the population coefficient β [30].

According to Ceballos [31], microeconometric studies often face potential endogeneity and address it with an instrumental–variables approach to obtain consistent estimators. However, obtaining valid instruments[9] can be challenging in practice. Even when instruments exist, they may be only weakly correlated with the endogenous regressors. Therefore, given possible endogeneity bias, we instrument prices and income because simultaneity or omitted variables could be present (see Appendix D).

We estimate the demand equation (Equation 12) using ordinary least squares (OLS), and we also implement two-stage least squares (2SLS) and generalized method of moments (GMM), see Table (1).

Regarding the estimates, the signs in the demand equation (Eq. 12) are as expected: an inverse relationship with prices and a direct relationship with income. In addition, to control for potential socio-demographic, educational, and household effects, we include relevant control variables in the equation. For a detailed description of other specifications, see Table (5).

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We compute the equivalent and compensating variation in income using the Vartia algorithm under several specifications. From models (1), (2), and (3), we obtain an equivalent variation of 1,2% and a compensating variation of 1,6% at the national level. To address potential endogeneity, we instrument household income in models (4) and (5), obtaining an equivalent variation of 1,4% and a compensating variation of 1,7%.

Estimation of Food Demand by Department In the national context, it is necessary to consider departmental heterogeneity when estimating food demand. The price and income variability characteristic of each department cannot be captured at the national level, making disaggregation crucial to obtain consistent results for each department.

From Table (2), results show smaller price effects in the country's central axis compared with the departments outside it, which may be due to a lower supply of foods and less developed transport and logistics infrastructure in those departments—factors that hinder market access and raise distribution costs. This observation supports prior evidence suggesting that food consumption patterns vary significantly across regions, as discussed in Borrega Reyes [32] and Rivero and Aliaga [33]. Consequently, it is advisable to estimate equivalent and compensating variation from a regional perspective.

Estimation of Food Demand by Income Deciles Studies often focus on the average effect of inflation on household income but are criticized for overlooking heterogeneity in outcomes. Several authors, however, have highlighted the differential effects of inflation on the distribution of household income and wealth, including Budd and Seiders [34], Wolff [35], Minarik [36], Fang et al. [37], Bourguignon et al. [38]. These authors emphasize the need to study price effects on inequality and income distribution.

It is therefore necessary to estimate demand in greater detail, paying attention to how prices affect food demand heterogeneously across income deciles. Given the public policy relevance of this issue, we estimate demand by income deciles in Table (3).

Estimation of Food Demand by Economic Activity Here we estimate food demand from the perspective of economic activity. Performance and output in each activity depend on various factors—such

[9] As Cameron and Trivedi [30] note, a valid instrument must satisfy: the relevance condition—there must be nonzero correlation between the instrument and the model's endogenous regressor ($corr(Z_i, X_i) \neq 0$); and the exogeneity condition—there must be no correlation between the instrument and the error term ($corr(Z_i, u_i) = 0$).

as income levels, sector type (primary, secondary, or tertiary), investment, and sector-specific government policies. In short, the behavior of different economic activities in Bolivia depends on a variety of internal and external factors. We therefore consider it prudent to estimate food demand separately by economic activity.

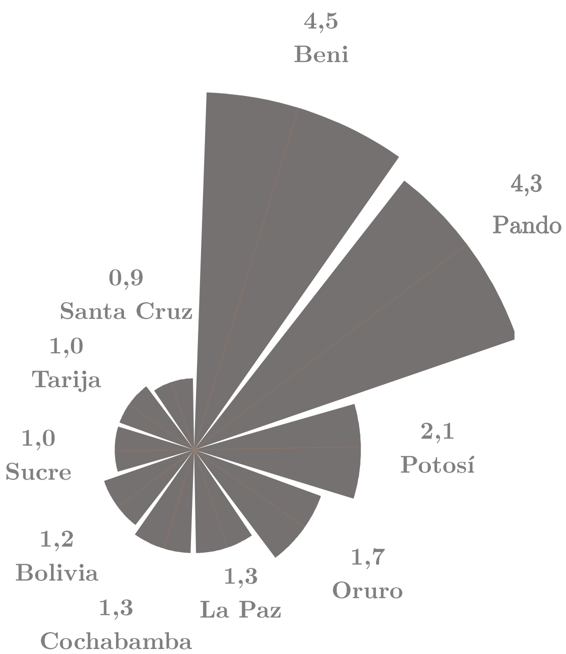
5.2 Application of the Vartia Algorithm

VER and VCR by department, income deciles, and economic activity. By department, parameters are statistically significant at the 5% level, and the signs are consistent with theory—negative for price and positive for income. Estimates show equivalent variation of 4,5% for Beni, 4,3% for Pando, and 2,1% for Potosí—the three departments with the largest income compensation. Conversely, Sucre at 1%, Tarija at 1%, and Santa Cruz at 0,9% record the smallest compensation (see Figure 12).

Compensating variation registers Pando at 2,9%, followed by Beni at 2% and Cochabamba at 1,9% as the highest compensation. By contrast, Oruro at 1,5%, Sucre at 1,3%, and Potosí at 1,1% record the lowest compensation.

These results are consistent with welfare theory: price increases directly affect households' consumption decisions, and maintaining the same level of welfare entails compensating income. For the Vartia algorithm,

Figure 11: VER and VCR by department



we group ten economic activities to examine income variation, including Trade, Manufacturing, Construction, Transportation and Storage, Education, Public Administration, Health, Agriculture, and the remainder. Parameters display the expected signs in the model and are statistically significant.

Figure 12: VER and VCR by department

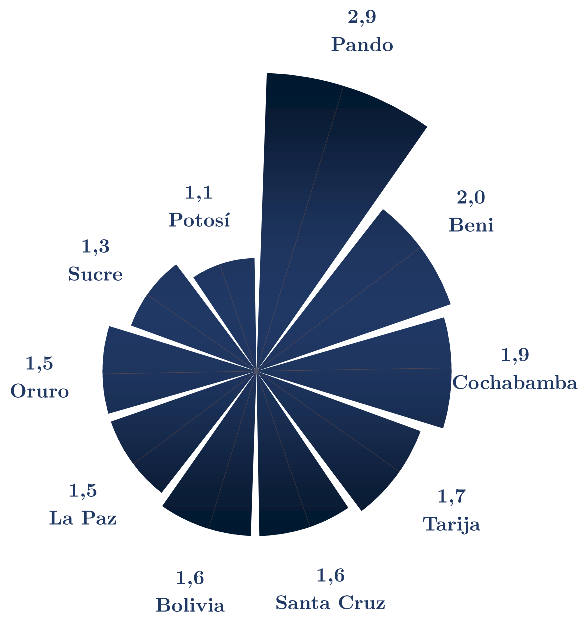
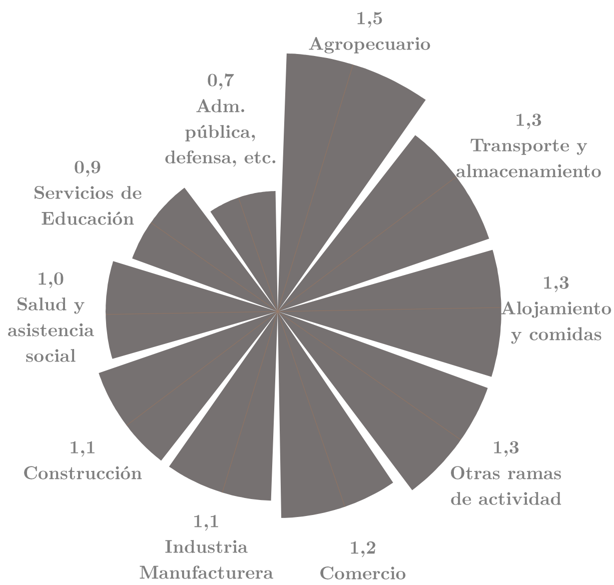


Figure 13: VER and VCR by economic activity



The algorithm yields an equivalent variation of 1,5% for Agriculture, followed by Transportation and Storage, Accommodation and Food Services, and the remainder of activities with 1,3% each—among the largest compensations. Among the smallest compensations are: Health and Social Assistance at 1%, Education at 0,9%, and Public Administration at 0,7% (see Figure 14).

Compensating variation records Transportation and

Figure 14: VER and VCR by economic activity

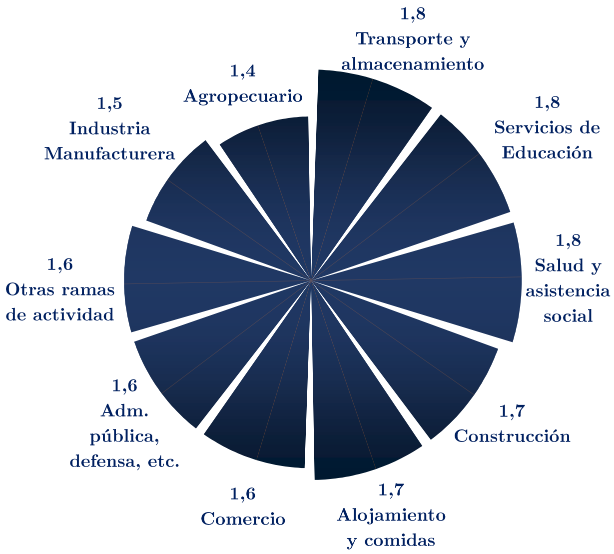
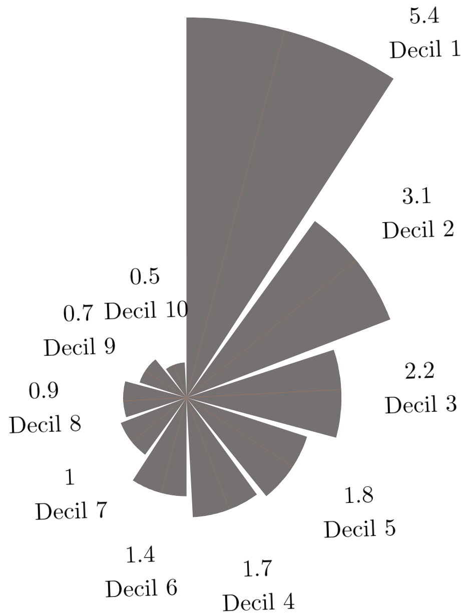


Figure 15: Equivalent variation of income by income deciles



Storage, Education, and Health at 1,8% each as the largest compensations, whereas Other Activities at 1,6%, Manufacturing at 1,5%, and Agriculture at 1,4% exhibit the smallest income compensations.

We apply the same estimation criteria across income deciles. In this case we report only equivalent variation, since the compensating variation results are not coherent with expectations: low-income consumers should require larger income compensation than higher-income deciles. Accordingly, equivalent variation for decile 1 is 5,4%, while decile 10 records 0,5% (see Figure 15). To benchmark these results, we refer to Rubin de Celis and Espinoza [15], who compute equivalent variation for the primary-education sector using two methods: integrability and the Vartia algorithm. Although both measure changes in individual welfare, the Vartia algorithm is more flexible, with the advantage of directly specifying the demand function. Among their key findings, a 1% decrease in the price of primary education would require a 10,2% income compensation to maintain welfare.

6 Conclusions and Recommendations

This research contributes to welfare economics in food and beverage consumption, an area where there has been a notable research lag at the national level. The results provide a coherent measure of welfare changes faced by individuals amid broad-based price increases.

These findings may serve as an important benchmark when setting the minimum wage—called the national minimum wage in Bolivia, Chile, Brazil, and Uruguay; the vital minimum wage in Argentina; the legal minimum wage in Colombia; the unified wage in Ecuador; and the vital minimum remuneration in Peru—since food expenditure carries a prominent weight in household consumption baskets.

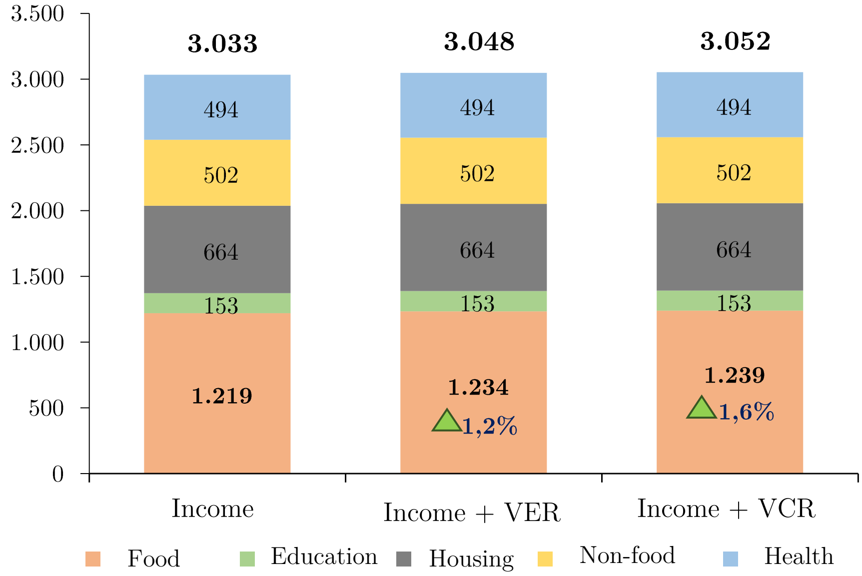
Likewise, this tool can serve as a useful reference for implementing differentiated wage-replacement policies by economic activity, income deciles, or even at the departmental level. Consequently, this research not only advances knowledge in microeconomics and food-consumption analysis but also has practical and potentially applicable implications for economic policy in general.

In Bolivia, VER and VCR for 2021 are 1,2% and 1,6%, respectively—percentages below the 2% national minimum wage increase in that year—reflecting a prudent and consistent adjustment by the national government.

Figure 16 presents a hypothetical exercise with a 1% increase in the general level of food prices. This would raise income allocated to food consumption, taking income from Bs1,219 to Bs1,234 for equivalent variation, and from Bs1,219 to Bs1,239 for compensating variation. In other words, this increase represents the monetary amount that would need to be compensated to keep household welfare unchanged in the face of higher prices.

Another aspect to consider is price stability in the country. Fuel subsidies, which allow setting price bands for urban, interdepartmental, and provincial transportation; price controls on certain staple foods

Figure 16: Compensated labor income, 2021



such as beef, chicken, and corn; and actions in education (e.g., caps on school tuition increases) all contribute to relatively low inflation, enabling the national government to implement wage adjustments consistent with population needs.

Moreover, since 2006, social policy—through government transfers to households, whether monetary or in kind—has helped offset price increases via higher non-labor income.

On the fiscal side, this measure contributes to public spending efficiency and safeguards the public finances when adjusting base pay in the public sector, especially in health and education.

Finally, for future research, we recommend using a demand system and considering other welfare-measurement algorithms. We also suggest analyzing implications for inequality and wage setting, as well as extending the welfare analysis to other dimensions such as housing, non-foods, health, and education.

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Table 1: Estimation of food demand, 2021

Quantity consumed	OLS			2SLS	GMM
Variable	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**
Prices 2021	-0.005308	-0.005056	-0.005544	-0.005517	-0.006311
	(0.000024)	(0.000024)	(0.000024)	(0.000025)	(0.000025)
	***	***	***	***	***
Household income	0.007657	0.007486	0.007485	0.001731	0.001804
	(0.00001)	(0.00001)	(0.00001)	(0.000028)	(0.000028)
	***	***	***	***	***
Central axis	0.470196		0.516036		
	(0.002317)		(0.002318)		
	***		***		
Experience	0.045378		0.025329	0.01948	0.020389
	(0.000179)		(0.000369)	(0.000198)	(0.000198)
	***		***	***	***
Experience squared	-0.00094		-0.000089	-0.000331	-0.000352
	(0.000002)		(0.000002)	(0.000003)	(0.000003)
	***		***	***	***
Higher education	-0.804556	-0.425292	-0.262675		
	(0.002329)	(0.002232)	(0.003313)		
		***	***		
Female		0.182094	0.152303		
		(0.002032)	(0.002051)		
		***	***		
Age		-0.00818	-0.029161		
		(0.000059)	(0.000342)		
		***	***		
Cannot read or write		-0.248145	-0.25384	-0.300221	-0.297049
		(0.003468)	(0.003986)	(0.00393)	(0.003926)
		***	***	***	***
Household members		0.646985	0.64072	0.710053	0.71106
		(0.000644)	(0.000663)	(0.000711)	(0.00071)
		***	***	***	***
Constant	4.369188	3.004203	3.010377	3.549676	3.637412
	(0.004448)	(0.006224)	(0.009986)	(0.007435)	(0.007413)
	***	***	***	***	***
Observations	249815	249815	249815	249815	249815
Coef. of determination	0.090484	0.103744	0.104513	0.056196	0.05737

Note 1: Standard errors are in parentheses. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (*) levels.
Note 2: The instruments used for income correspond to human capital variables, gender, and geographical characteristics. To instrument prices and income, alternatives such as geographical location, expenditure on food away from home, precipitation, temperature and agricultural yields (by department) were used, as well as prices of imported, domestic and hybrid products.
Source: Authors' own elaboration

Table 2: Estimation of demand by department

Variable	Bolivia	Sucre	La Paz	Cbba.	Oruro	Potosí	Tarija	Santa Cruz	Beni	Pando
Prices 2021	-0.00669	-0.006553	-0.010221	-0.004333	-0.021828	-0.034361	0.007249	-0.003243	-0.092038	-0.072996
Std. errors	(0.0007)	(0.0015)	(0.0027)	(0.0022)	(0.0028)	(0.0021)	(0.0017)	(0.0009)	(0.0101)	(0.0168)
t-statistic	-9.03	-4.42	-3.81	-2	-7.87	-16.31	4.39	-3.46	-9.14	-4.34
p-value	***	***	***	**	***	***	***	***	***	***
Household income	0.007286	0.005879	0.007631	0.007779	0.005997	0.005375	0.006836	0.008013	0.004736	0.011387
Std. errors	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0002)	(0.0007)
t-statistic	136.53	39.97	92.24	63.49	50.55	43.06	41.29	75.05	23.18	15.44
p-value	***	***	***	***	***	***	***	***	***	***
Constant	4.905298	4.363437	4.902954	4.89171	5.601239	6.802569	3.519836	4.688511	15	15
Std. errors	(0.0809)	(0.1671)	(0.2788)	(0.2391)	(0.289)	(0.2293)	(0.1811)	(0.1116)	(1.009)	(1.6757)
t-statistic	60.63	26.12	17.58	20.46	19.38	29.67	19.43	42	14.7	8.66
p-value	***	***	***	***	***	***	***	***	***	***
Observations	249815	17029	63363	44779	18199	15934	21784	47412	12235	9080
Coef. of determination	0.069729	0.087512	0.118505	0.082705	0.123564	0.123044	0.072703	0.106595	0.046299	0.025622

Note: Standard errors are in parentheses. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (*) levels.
Source: Authors' own elaboration

Table 3: Estimation of demand by income deciles

Variable	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
Prices 2021	-0.012917	-0.010002	-0.00882	-0.007359	-0.012494	-0.008154	-0.003693	-0.00601	-0.002705	-0.001921
Std. errors	(0.0019)	(0.0018)	(0.0016)	(0.0014)	(0.0015)	(0.0013)	(0.0012)	(0.0014)	(0.0014)	(0.0012)
t-statistic	-6.9	-5.49	-5.44	-5.11	-8.25	-6.07	-2.96	-4.26	-1.98	-1.56
p-value	***	***	***	***	***	***	***	***	**	*
Household income	0.018214	0.018695	0.016738	0.015547	0.013376	0.013108	0.010866	0.010642	0.008587	0.004561
Std. errors	(0.0021)	(0.0009)	(0.0007)	(0.0007)	(0.0006)	(0.0007)	(0.0004)	(0.0004)	(0.0004)	(0.0002)
t-statistic	8.76	20.25	25.16	23.4	22.27	19.73	25.21	30.23	24.16	21.36
p-value	***	***	***	***	***	***	***	***	***	***
Constant	4.477287	4.283053	4.418191	4.254382	4.916943	4.487441	3.966729	4.385582	4.107135	4.673144
Std. errors	(0.2132)	(0.2079)	(0.1874)	(0.1715)	(0.1896)	(0.1765)	(0.1577)	(0.1746)	(0.1775)	(0.1673)
t-statistic	21	20.6	23.58	24.8	25.94	25.42	25.15	25.12	23.13	27.93
p-value	***	***	***	***	***	***	***	***	***	***
Observations	16960	20567	23023	24401	24880	26123	27327	28301	28685	29548
Coef. of determination	0.045478	0.077563	0.0647	0.089441	0.08318	0.072139	0.106671	0.089737	0.093331	0.069499

Note: Standard errors are in parentheses. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (*) levels.
Source: Authors' own elaboration

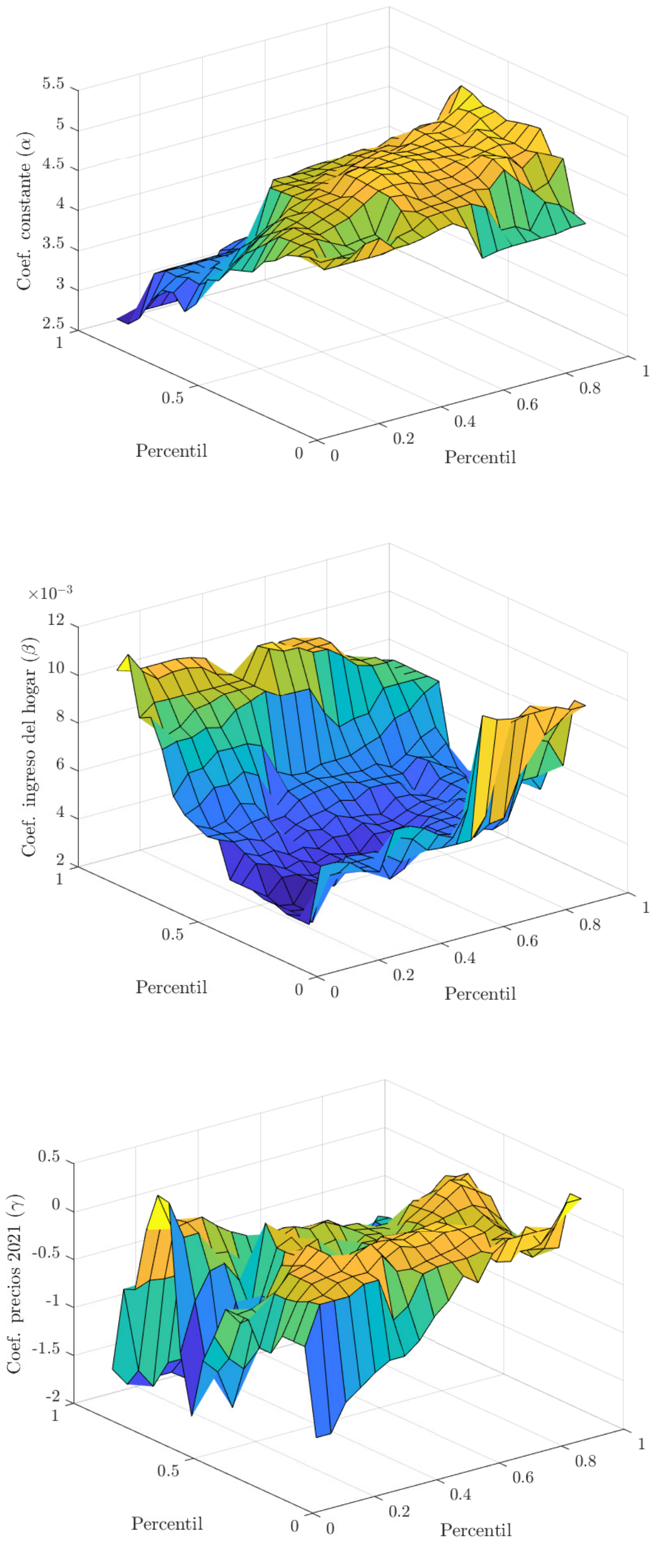
Table 4: Estimación de la demanda de alimentos por actividad económica

Variable	Comercio	Manuf.	Constr.	Transporte	Aloj. y com.	Educación	Adm. Púb.	Salud	Agropecuario
Precios 2021	-0.003217	-0.003106	-0.003537	-0.012239	-0.003895	-0.007982	-0.003241	-0.011942	-0.003645
Errores est.	(0.0011)	(0.0012)	(0.0016)	(0.0016)	(0.0015)	(0.0019)	(0.0018)	(0.0033)	(0.0022)
Estadístico-t	-2.81	-2.67	-2.21	-7.75	-2.54	-4.17	-1.76	-3.64	-1.67
Valor-p	***	***	***	***	***	***	*	***	*
Ing. Hogar	0.006358	0.006931	0.008978	0.009807	0.007643	0.007231	0.006299	0.005934	0.006074
Errores est.	(0.0006)	(0.0003)	(0.0005)	(0.0006)	(0.0005)	(0.0007)	(0.0005)	(0.0008)	(0.0007)
Estadístico-t	10.52	22.05	16.95	17.02	14.08	10.83	13.98	7.44	8.47
Valor-p	***	***	***	***	***	***	***	***	***
Constante	4.79158	4.455405	4.451277	5.398812	5.044142	5.163519	4.220152	5.802259	4.497852
Errores est.	(0.1672)	(0.1495)	(0.1981)	(0.1984)	(0.198)	(0.2528)	(0.2402)	(0.4222)	(0.3034)
Estadístico-t	28.65	29.8	22.47	27.21	25.48	20.43	17.57	13.74	14.83
Valor-p	***	***	***	***	***	***	***	***	***
Obervaciones	35991	28348	26050	24242	25013	10450	12476	7912	7093
Coef. Deter.	0.0591	0.0711	0.0935	0.0853	0.0756	0.0747	0.0657	0.0391	0.0519

Nota: Los errores estándar se encuentran entre paréntesis. Los asteríscos son niveles de significancia estadística al 1% (***), 5% (**) y 10% (*)
Elaboración: Propia de los autores

A Quantile Regression of Food Demand

Figure 17: Estimated coefficients of food demand, 2021



B Food Demand Estimates

Table 5: Estimation of food demand, 2021

Variable	Mod. 1	Mod. 2	Mod. 3	Mod. 4	Mod. 5	Mod. 6	Mod. 7	Mod. 8	Mod. 9	Mod. 10	Mod. 11
Prices 2021	-0.00498	-0.00531	-0.00493	-0.00506	-0.00554	-0.00558	-0.00555	-0.00548	-0.00506	-0.00673	-0.00722
	(0.000024)	(0.000024)	(0.000024)	(0.000024)	(0.000024)	(0.000024)	(0.000024)	(0.000024)	(0.000024)	(0.000459)	(0.000448)
Household income	0.007631	0.007657	0.007727	0.007486	0.007485	0.007487	0.007485	0.007416	0.007483	0.002048	0.002138
	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.000511)	(0.00051)
Central axis		0.470196			0.516036	0.520867	0.507527				
		(0.002317)			(0.002318)	(0.002312)	(0.002316)				
Experience		0.045378			0.025329					0.01517	0.015195
		(0.000179)			(0.000369)					(0.003667)	(0.003664)
Exp. 2		-0.00094			-8.9E-05					-0.0003	-0.0003
		(0.000002)			(0.000002)					(0.000051)	(0.00005)
Higher educ.		-0.80456	-0.68821	-0.42529	-0.26268	-0.37215	-0.41824	-0.44107			
		(0.002329)	(0.002216)	(0.002232)	(0.003313)	(0.002254)	(0.002243)	(0.002217)			
Female			-0.24625	0.182094	0.152303	0.190698	0.141462	0.128264	0.185516		
			(0.002016)	(0.002032)	(0.002051)	(0.002431)	(0.002079)	(0.00203)	(0.002039)		
Age			-0.02107	-0.00818	-0.02916	-0.00247	-0.00864	-0.0049	-0.00752		
			(0.000056)	(0.000059)	(0.000342)	(0.000071)	(0.000065)	(0.000059)	(0.000065)		
Cannot read or write				-0.24815	-0.25384	-0.18827	-0.18218	-0.09357	-0.20452	-0.36673	-0.37016
				(0.003468)	(0.003986)	(0.003492)	(0.003476)	(0.00348)	(0.006395)	(0.072992)	(0.072914)
Household members				0.646985	0.64072	0.652088	0.648268	0.647874	0.647837	0.706626	0.709003
				(0.000644)	(0.000663)	(0.000722)	(0.000643)	(0.000638)	(0.000646)	(0.013125)	(0.01309)
Marital status						*Base category: single*					
Married						-0.18641					
						(0.003328)					
Cohabiting						0.273202					
						(0.003433)					
Separated						-0.02448					
						(0.003936)					
Divorced						-0.19748					
						(0.006587)					
Widowed						-0.28138					
						(0.003942)					
Labour status						*Base category: employed*					
Unemployed							0.275872				
							(0.006851)				
Job seeker							1.866805				
							(0.032357)				
Temporary							0.035927				
							(0.002921)				
Permanent							0.359116				
							(0.007266)				
Department						*Base category: Sucre*					
La Paz								0.586686			
								(0.003755)			
Cochabamba								1.29082			
								(0.004266)			
Oruro								-0.26608			
								(0.004548)			
Potosí								-0.65408			
								(0.00411)			
Tarija								1.133043			
								(0.004899)			
Santa Cruz								1.152686			
								(0.003895)			
Beni								1.28396			
								(0.007308)			
Pando								5.250261			
								(0.031878)			
Education level						*Base category: none*					
Primary									0.133809		
									(0.011402)		
Secondary									0.016848		
									(0.006792)		
Higher									0.085478		
									(0.007185)		
Others									-0.3631		
									(0.007354)		
Constant	4.681978	4.369188	6.154135	3.004203	3.010377	2.356697	2.662217	1.977785	2.86278	3.767986	3.827529
	(0.003283)	(0.004448)	(0.005038)	(0.006224)	(0.009986)	(0.006893)	(0.006678)	(0.007232)	(0.014106)	(0.136289)	(0.135751)
N	249815	249815	249815	249815	249815	249815	249815	249815	249815	249815	249815
Coef. Det.	0.085783	0.090484	0.088641	0.103744	0.104513	0.104913	0.104553	0.11139	0.103754	0.05159	0.052735

Note: Standard errors are in parentheses. Coefficients are significant at the 5% level.
Source: Authors' own elaboration

C Division of products

Table 6: Breakdown of products used in the sample

No.	Product
1	Regular bread
2	Special bread
3	Biscuits/cookies
4	Bakery products (cake, baked empanadas, sponge cake, cuñapé, etc.)
5	Fried pastry products
6	Rice
7	Maize (corn)
8	Quinoa
9	Pasta (noodles)
10	Flour (wheat, maize, etc.)
11	Other cereals (oats, flaked cereals, etc.)
12	Boneless beef (special cuts)
13	Bone-in beef (with sinew, second/third quality)
14	Ground beef (regular/special)
15	Chicken meat (whole, cut-up)
16	Fresh pork, whole or special cuts
17	Fresh sheep meat by pieces (lamb)
18	Fresh llama meat
19	Sausages and cold cuts (sausages, chorizo, deli meats, etc.)
20	Offal from beef, lamb, pork, chicken (liver, heart, head, etc.)
21	Charque, chalona (dried meat of any animal)
22	Fresh fish (sábalo, silverside, trout, surubí, pacú, etc.)
23	Canned or processed fish and seafood (sardines, tuna, etc.)
24	Fluid milk
25	Powdered milk
26	Yogurt
27	Other dairy products
28	Cheese
29	Non–cow's milk products (soy milk)
30	Eggs
31	Cooking oil
32	Butter
33	Lard, margarine
34	Plantain
35	Apple
36	Papaya
37	Mandarin
38	Orange
39	Grape
40	Peach
41	Watermelon
42	Other fruits: pineapple, lemon, mango, pear, including canned, etc.
43	Tomato
44	Onion
45	Carrot
46	Lettuce
47	Fresh corn on the cob
48	Other vegetables (squash, green beans, bell pepper, etc.)
49	Mixed chopped vegetables / assorted legumes in bags
50	Potato
51	Cassava/manioc
52	Dried tubers (chuño, tunta)
53	Dry legumes (beans)
54	Lentils
55	Peanuts
56	Prepared/processed products (soaked chuño, ground peanuts, canned peas, etc.)
57	Other oilseed products (chia, amaranth, olives, soy, etc.)
58	Granulated sugar
59	Jams and jellies
60	Honey, cane honey (cane syrup)
61	Chocolate
62	Candies/sweets, chewing gum
63	Artificial sweeteners, various sweeteners, chancaca, etc.
64	Salt
65	Dried chili peppers (pods)
66	Spices, sauces, seasonings, dressings and similar
67	Coffee
68	Tea
69	Coca leaf
70	Chocolate-flavored powders (Toddy, Chocolike, etc.)
71	Natural herbs (chamomile, eucalyptus, boldo, lemon verbena, etc.)
72	Soft drink in bottle/can
73	Fruit and vegetable juices by the glass, bottled/carton juices, energy drinks
74	Bottled water
75	Wine, beer, spirits (singani, whisky), tobacco (cigarettes), etc.

Source: Authors' own elaboration

D Estimation Using Instrumental Variables

The endogeneity problem was addressed by including appropriate instruments, considering relevant household characteristics. Variables that are uncorrelated with the error term and influence prices were selected, such as agricultural yield, precipitation, temperature, and prices of imported, domestic, and hybrid products. Likewise, the relevance of the instruments used in the analysis was confirmed.

Table 7: Estimation of food demand using instrumental variables

Quantity consumed	**Estimation using instrumental variables (2SLS)**						
Variable	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Household income	0.001731	0.009421	0.005054	0.010989	0.005602	0.008509	0.009026
	(0.000028)	(0.000021)	(0.000013)	(0.000022)	(0.000026)	(0.000012)	(0.000012)
Prices 2021	-0.005517	-0.007772	-0.008827	-0.007180	-0.009273	-0.008437	-0.008359
	(0.000025)	(0.000026)	(0.000025)	(0.000026)	(0.000027)	(0.000025)	(0.000025)
Experience	0.019480	-0.003921	0.009391	-0.008660	0.007621	-0.001083	-0.002651
	(0.000198)	(0.000187)	(0.00018)	(0.000188)	(0.000191)	(0.000179)	(0.00018)
exp2	-0.000331	0.000013	-0.000183	0.000082	-0.000157	-0.000029	-0.000006
	(0.000003)	(0.000003)	(0.000003)	(0.000003)	(0.000003)	(0.000003)	(0.000003)
Exp. squared	-0.300221	-0.012804	-0.176930	0.045883	-0.155795	-0.047622	-0.028250
	(0.00393)	(0.003719)	(0.003702)	(0.003737)	(0.003783)	(0.003656)	(0.003657)
Household members	0.710053	0.635704	0.677898	0.620481	0.672797	0.644414	0.639397
	(0.000711)	(0.000657)	(0.000656)	(0.000659)	(0.000673)	(0.000641)	(0.000641)
Constant	3.549676	2.542634	3.365256	2.225310	3.321839	2.763360	2.671076
	(0.007435)	(0.007015)	(0.006119)	(0.007017)	(0.007579)	(0.005941)	(0.005941)
Observations	251780	251780	251780	251780	251780	251780	251780
Coef. of determination	0.05620	0.09698	0.09487	0.08418	0.09815	0.10108	0.09902
Instruments	Experience Female Central axis Higher educ. Prices 2020	Experience Female Central axis Higher educ. Prices 2020 Imp. dom prices	Experience Female Central axis Higher educ. Prices 2020 Inflation 2021	Experience Female Central axis Higher educ. Prices 2020 Imp. dom prices Agric. yield	Experience Female Central axis Higher educ. Prices 2020 Precipitation Temperature	Experience Female Central axis Higher educ. Prices 2020 Inflation Imp. dom prices	Experience Female Central axis Higher educ. Prices 2020 Inflation 2021 Imp. dom prices Var. agric. yield
Endogeneity test							
Robust score chi2	40954.6	283444	438881	304911	274371	438867	445608
Robust regression F	40926.6	149733	231008	161386	144205	232017	235650
Relevance of excluded exogenous variables: adjusted partial R-2							
Prices 2021	-	0.8860	0.9635	0.8868	0.8739	0.9640	0.9640
Household income	0.0200	0.0201	0.0355	0.0205	0.0147	0.0460	0.0465
Overidentification test of restrictions							
Chi2(2) value	53250	283286	87691.1	610538	621739	300907	475734

Note 1: Regarding the price instruments, prices of imported, domestic, and hybrid products were considered, as well as 2021 price inflation and previous-period prices. Agricultural yield was approximated using agricultural production and cultivated area per agricultural year (in hectares). Likewise, average precipitation (in millimeters) and temperature (in degrees Celsius) by department were used.
Note 2: To determine the exogeneity of the regressors in the model, endogeneity tests were performed, specifically the Durbin and Wu–Hausman tests, after the 2SLS estimation. The results yielded significant test statistics in all cases, indicating that the variables should be considered endogenous and therefore must be instrumented. In addition, the explanatory power of the instruments used was evaluated; they must be correlated with the endogenous regressors included, but not with the error term. In all cases, the instruments were found to be significant, indicating their ability to explain the variation in the endogenous variables.
Note 3: Standard errors are in parentheses. Coefficients are significant at the 5% level.
Source: Authors' own elaboration

E Inflation by Divisions and Weights

Table 8: Incidence of inflation 2020–2022 (In percent and percentage points)

Weight	Division	Dec-20	Dec-21	Dec-22
100.00	***Consumer Price Index***	***0.67***	***0.90***	***3.12***
27.06	Food and Non-Alcoholic Beverages	-0.21	0.13	1.76
0.88	Alcoholic Beverages and Tobacco	0.00	0.00	0.00
7.56	Clothing and Footwear	-0.11	0.01	0.09
8.56	Housing and Basic Services	0.01	0.02	0.09
6.08	Furniture, Household Goods and Domestic Services	0.03	0.04	0.30
3.55	Health	0.27	0.04	0.04
9.07	Transport	0.15	0.21	0.19
5.43	Communications	0.08	0.07	-0.07
6.22	Recreation and Culture	0.13	0.09	-0.03
4.07	Education	0.12	0.27	0.22
13.95	Food and Beverages Consumed Away from Home	0.19	0.10	0.35
7.55	Miscellaneous Goods and Services	0.00	-0.07	0.17

Source: Authors' own elaboration
Source: National Institute of Statistics

F Recovering Utility from the Demand Function

Next, we proceed to recover the utility function from which the linear demand function used in this study is derived. The procedure follows Border [24].

Given a demand function x^*:

1. Considering the following differential equation:

$$\frac{\partial \mu(p)}{\partial p_i} = x_i^*(p, \mu(p)) \tag{13}$$

The explicit solution is written in terms of the initial condition $\mu(p^0) = m^0$ as $\mu(p; p^0, m^0)$.

2. Next, fixing a price vector p^*, we define an indirect utility function v:

$$v(p, m) = \mu(p^*; p, m) \tag{14}$$

3. We find the inverse of the demand function for (p, m) as a function of x^*.
4. Define utility on the range of x^* as:

$$U(x) = \mu(p^*; p, m) \tag{15}$$

However, this raises questions such as whether the differential equation has a solution, or whether the utility function U generates the demand function x^*. To explore this, we consider two goods x and y. By the homogeneity of x^*, we can take $p_y = 1$, so that the price of x is simply denoted as p.
Starting from a Cobb–Douglas utility function:

$$u(x, y) = x^\alpha y^\beta \tag{16}$$

where $\alpha + \beta = 1$, the demand functions are:

$$x^*(p, m) = \frac{\alpha m}{p}, \quad y^*(p, m) = \beta m \tag{17}$$

The indirect utility function is expressed as:

$$v(p, m) = m\beta^\beta \left(\frac{\alpha}{p}\right)^\alpha \tag{18}$$

Meanwhile, the expenditure function is given by:

$$e(p, v) = v\beta^{-\beta} \left(\frac{p}{\alpha}\right)^\alpha \tag{19}$$

Considering the points (p^0, m^0), we define:

$$\mu(p; p^0; m^0) = e(p; v(p^0, m^0)) = m^0 \left(\frac{p}{p^0}\right)^\alpha \quad (20)$$

Evaluating at $p = p^0$, we have:

$$\mu(p^0; p^0; m^0) = m^0 \quad (21)$$

For each point (p^0, m^0), the function $\mu(p) = \mu(p; p^0, m^0)$ satisfies the ordinary differential equation:

$$\frac{\partial \mu}{\partial p} = \alpha[m^0 (p^0)^\alpha] p^{\alpha - 1} = \frac{\alpha \mu(p)}{p} = x^*(p, \mu(p)) \quad (22)$$

Recovering the Utility Function: Let $n = 2$ (x_1 and x_2) and $p_2 = 1$, so there is effectively only one price p and one differential equation (for x_1):

$$\mu'(p) = x(p, \mu(p)) \quad (23)$$

Where:

$$x(p, m) = \frac{\alpha m}{p} \quad (24)$$

Here, x is the demand for x_1. From the budget constraint, we can infer:

$$x_2 = (1 - \alpha) m \quad (25)$$

Rewriting in terms of y and x, $y' = \alpha y / x$. Integrating both sides gives:

$$\ln(\mu) = \alpha \ln(p) + C \quad (26)$$

Then:

$$u(x_1, x_2) = \omega(p, m) \quad (27)$$

After simplification, we obtain:

$$u(x_1, x_2) = c\, x_1^\alpha x_2^{1-\alpha} \quad (28)$$

where $c = (1 - \alpha)^{(1-\alpha)} \alpha^\alpha$, which is a Cobb–Douglas utility function.

Existence of Solutions for Total Differential Equations: Given an open set $A \times B \subset \mathbb{R}^n \times \mathbb{R}$ with a typical element (p, m) and a function

$$\xi : A \times B \to \mathbb{R}^n,$$

a function $\mu : A \to B$ is a local solution of the total differential equation:

$$M' = \xi(p, M) \quad (29)$$

over $U \subset A$ if:

$$\mu'(p) = \xi_i(p, \mu(p)) \qquad \forall p \in U \quad (30)$$

The total differential equation can be written as a system of partial differential equations:

$$\frac{\partial M}{\partial p_i} = \xi_i(p, \mu(p)) \quad (31)$$

This equation is *completely integrable* if, for each $(p^0, m^0) \in A \times B$, there exists a neighborhood U of p^0 and a unique continuously differentiable function $\mu : U \to B$ (depending on (p^0, m^0)) that satisfies (30) and the initial condition:

$$\mu(p^0) = m^0 \quad (32)$$

Integrability of Demand Proof of Theorem 1: Given a function ξ, the budget exhaustion condition (B) implies the condition of the Hurwicz–Uzawa existence theorem. Hence, all the assumptions of that theorem are satisfied, and therefore, there exists a unique function:

$$\mu : \mathbb{R}^n_{++} \times \mathbb{R}^n_{++} \times \mathbb{R}_+ \to \mathbb{R} \quad (33)$$

such that, for each initial condition (p^0, m^0), $\mu(\cdot; p^0, m^0)$ satisfies (33) and $\mu(p^0; p^0, m^0) = m^0$, and such that $\mu(p; \cdot, \cdot)$ is continuous for each p.

It is observed that while different initial conditions (p^0, m^0) may or may not define different functions $\mu(\cdot; p^0, m^0)$, the representations of these functions do not intersect.

Consider two different initial conditions (p^0, m^0) and (p^1, m^1). Either they define the same function μ, or one lies above the other. That is:

$$\mu(\cdot; p^0, m^0) = \mu(\cdot; p^1, m^1), \quad \mu(\cdot; p^0, m^0) < \mu(\cdot; p^1, m^1), \quad \mu(\cdot; p^0, m^0) > \mu(\cdot; p^1, m^1) \tag{34}$$

By the Global Existence Theorem, there exists a unique function satisfying differential equation (31) through the point $(\bar{p}, \bar{m})$. Hence:

$$\mu(\cdot; p^0, m^0) = \mu(\cdot; p^1, m^1) = \mu(\cdot; \bar{p}, \bar{m}) \tag{35}$$

Furthermore, suppose for some $\bar{p}$ we have $\mu(\bar{p}; p^0, m^0) > \mu(\bar{p}; p^1, m^1)$, and likewise for $\mu(\bar{p}; p^0, m^0) < \mu(\bar{p}; p^1, m^1)$. Since μ is continuous in its first variable, by the Intermediate Value Theorem there exists $0 < \lambda < 1$ such that:

$$\mu\left((1-\lambda)\bar{p} + \lambda\tilde{p}; p^0, m^0\right) = \mu\left((1-\lambda)\bar{p} + \lambda\tilde{p}; p^1, m^1\right) \tag{36}$$

which implies:

$$\mu\left(\cdot; p^0, m^0\right) = \mu\left(\cdot; p^1, m^1\right) \tag{37}$$

If indeed ξ is derived from utility maximization, then we know that its income-compensating function satisfies the differential equation (30). From the uniqueness part of the existence theorem, we find that ω is an indirect utility function, and the constructed u is a utility function. (Different choices of p^* yield different utility functions.) However, since ξ is not necessarily known to be a demand function, it is assumed that this u satisfies the Weak Axiom of Revealed Preference [39].